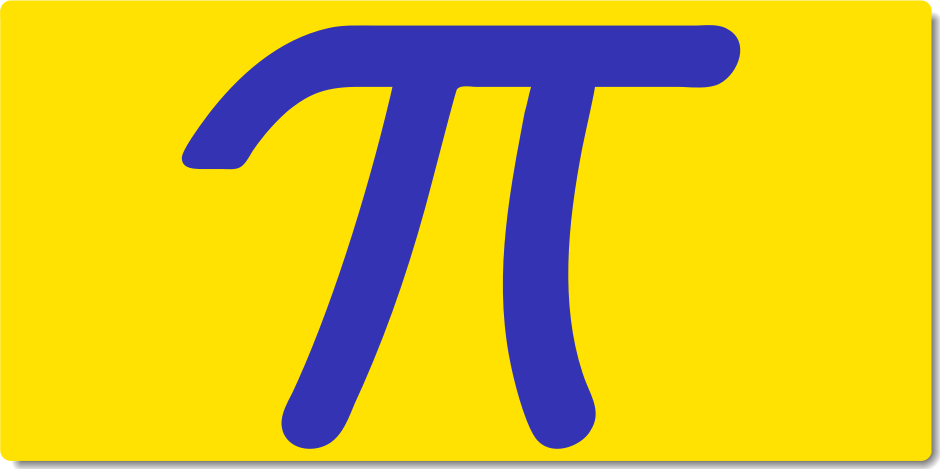


Ottavio Caligaris

Università di Genova - Polo di Savona - Facoltà di Ingegneria

The Indiana Pi Bill, 1897
ENGROSSED HOUSE BILL No. 246 A Bill for an act introducing a new mathematical truth and offered as a contribution to education to be used only by the State of Indiana free of cost by paying any royalties whatever on the same, provided it is accepted and adopted by the official action of the Legislature of 1897.

Section 1 Be it enacted by the General Assembly of the State of Indiana: It has been found that a circular area is to the square on a line equal to the quadrant of the circumference, as the area of an equilateral rectangle is to the square on one side. The diameter employed as the linear unit according to the present rule in computing the circle's area is entirely wrong, as it represents the circle's area one and one-fifth times the area of a square whose perimeter is equal to the circumference of the circle. This is because one fifth of the diameter fails to be represented four times in the circle's circumference. For example: if we multiply the perimeter of a square by one-fourth of any line one-fifth greater than one side, we can in like manner make the square's area to appear one-fifth greater than the fact, as is done by taking the diameter for the linear unit instead of the quadrant of the circle's circumference.

Section 2 It is impossible to compute the area of a circle on the diameter as the linear unit without trespassing upon the area outside of the circle to the extent of including one-fifth more area than is contained within the circle's circumference, because the square on the diameter produces the side of a square which equals nine when the arc of ninety degrees equals eight. By taking the quadrant of the circle's circumference for the linear unit, we fulfill the requirements of both quadrature and rectification of the circle's circumference. Furthermore, it has revealed the ratio of the chord and arc of ninety degrees, which is as seven to eight, and also the ratio of the diagonal and one side of a square which is as ten to seven, disclosing the fourth important fact, that the ratio of the diameter and circumference is as five-fourths to four; and because of these facts and the further fact that the rule in present use fails to work both ways mathematically, it should be discarded as wholly wanting and misleading in its practical applications.

Section 3 In further proof of the value of the author's proposed contribution to education and offered as a gift to the State of Indiana, is the fact of his solutions of the trisection of the angle, duplication of the cube and quadrature of the circle having been already accepted as contributions to science by the American Mathematical Monthly, the leading exponent of mathematical thought in this country. And be it remembered that these noted problems had been long since given up by scientific bodies as insolvable mysteries and above man's ability to comprehend.

Il testo riporta il contenuto del
Indiana House Bill No. 246, 1897, noto come Indiana pi bill.
http://www.agecon.purdue.edu/crd/Localgov/Second%20Level%20pages/indiana_pi_bill.htm

Verso la fine della sezione 2 dice semplicemente che
"The ratio of the diameter and circumference is as five-fourths to four,"
il che significa

$$\pi = \frac{16}{5} = 3.2.$$

La sezione prosegue criticando le i valori fino ad allora attribuiti a π come "wholly wanting and misleading." Cioè totalmente carenti e fuorvianti.

Il Dr. Edwin J. Goodwin, M.D., era un medico che si dilettava di matematica nel 1894 pubblico' un articolo dal titolo

QUADRATURE OF THE ClRCLE

su: The American Mathematical Monthly, Vol. 1, No. 7 (July 1894), pp. 246-248
https://www.agecon.purdue.edu/crd/localgov/Topics/Materials/Pi_Goodwin_AmMathMonthly_1894.pdf
Nel 1897 scrisse una proposta di legge che conteneva i suoi risultati e convinse il suo rappresentante di stato,

Taylor I. Record , a presentarla alla Indiana General Assembly.
La proposta fu rinviata a tempo indeterminato anche grazie all'intervento del Prof. C.A.Waldo
https://www.agecon.purdue.edu/crd/localgov/Topics/Essays/Pi_Bill_Indiana_1897.htm

LEGISLATIVE HISTORY

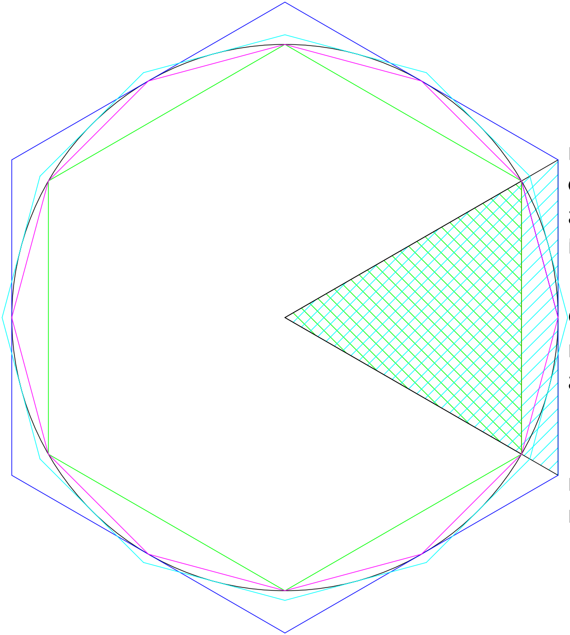
Introduced
IN THE HOUSE
Read first time January 18th, 1897
Referred to Committee on Canals
Reported and referred to Committee on Education January 19th, 1897
Reported back February 2nd, 1897
Read second time February 5th, 1897
Ordered engrossed February 5th, 1897
Read third time February 5th, 1897
Passed February 5th, 1897
Ayes - 67 - Noes -0-
Introduced by Record
IN THE SENATE
Read first time and referred to
committee on Temperance, February 11th, 1897
Reported favorable February 12th, 1897
Read second time and indefinitely postponed February 12, 1897

3

Archimede trovò una approssimazione di π utilizzando i perimetri dei poligoni regolari inscritti e circoscritti alla circonferenza unitaria.

- Archimede usò un poligono di $3 * 2^5 = 96$ lati ottenendo 2 cifre decimali (287-212 B.C.)
- Liu-Hui usò un poligono di $3 * 2^{18} = 3072$ lati ottenendo 5 cifre decimali (225-295 A.D.)
- Al-Kashi usò un poligono di $3 * 2^{54} = 3805\,306\,368$ lati ottenendo 14 cifre decimali (1380-1429 A..D.)
- Ludolph van Ceulen usò un poligono di $3 * 2^{124} = 27\,670\,116\,110\,564\,327\,424$ lati ottenendo 35 cifre decimali (1540-1610 A.D.)

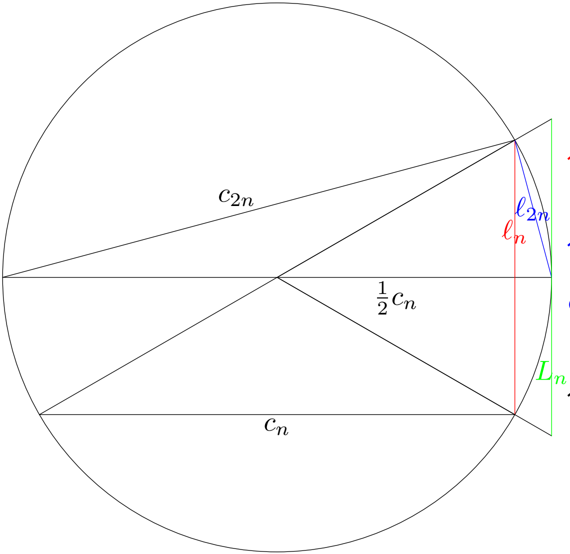
https://math.dartmouth.edu/archive/m56s13/public_html/Han_proj.pdf



L'area dei poligoni regolari inscritti nel cerchio di raggio 1 approssima per difetto l'area del cerchio.

Quella dei poligoni regolari circoscritti la approssima per difetto.

L'approssimazione migliora se raddoppia il numero dei lati



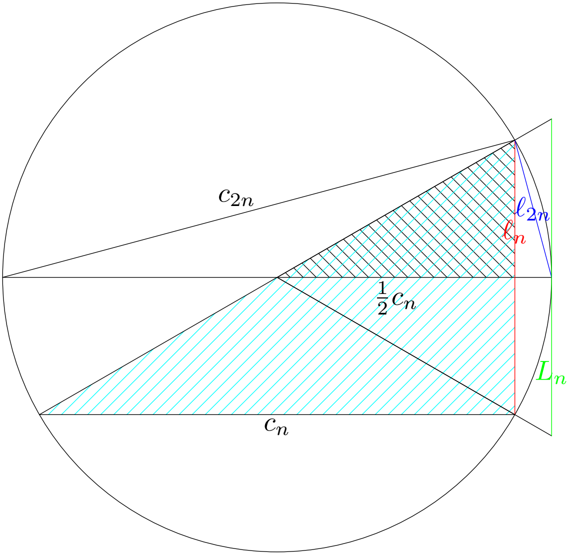
L_n lato del poligono regolare circoscritto di n lati

ℓ_n lato del poligono regolare inscritto di n lati

ℓ_{2n},L_{2n} lato del poligono regolare inscritto, circoscritto di $2n$ lati

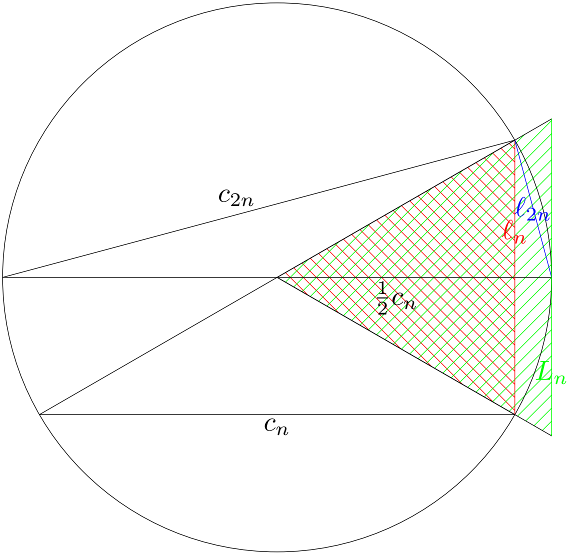
c_n, c_{2n} complementi di ℓ_n, ℓ_{2n}

c_n, ℓ_n ed il diametro per un estremo di ℓ_n formano un triangolo rettangolo.



La similitudine dei triangoli assicura che il segmento che congiunge il centro del cerchio ed il punto medio di ℓ_n misura

$$\frac{1}{2}c_n$$



Inoltre , per la similitudine dei triangoli tratteggiati in figura si ha

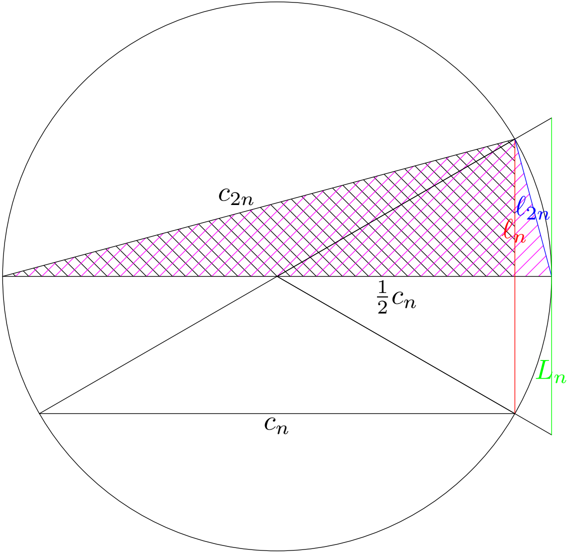
$$\frac{\ell_n}{L_n} = \frac{1}{2} c_n$$

da cui

$$c_n = \frac{2\ell_n}{L_n}$$

ed anche

$$c_{2n} = \frac{2\ell_{2n}}{L_{2n}}$$



Per la similitudine dei triangoli tratteggiati in figura si ha

$$\frac{c_{2n}}{2} = \frac{\frac{\ell_n}{2}}{\ell_{2n}}$$

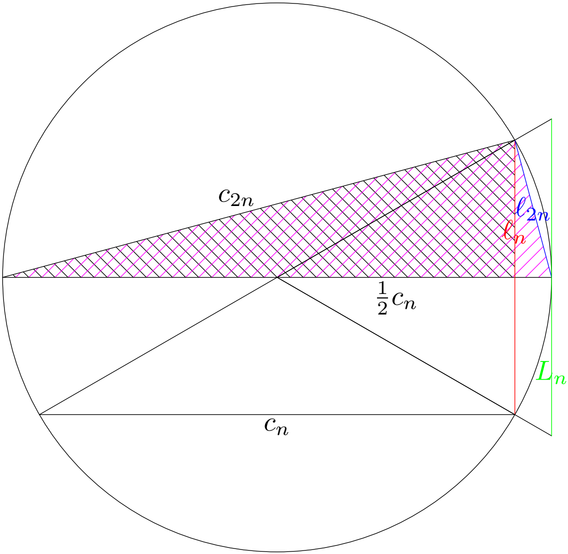
$$c_{2n} = \frac{\ell_n}{\ell_{2n}}$$

Pertanto

$$\frac{2\ell_{2n}}{L_{2n}} = c_{2n} = \frac{\ell_n}{\ell_{2n}}$$

da cui

$$2\ell_{2n}^2 = \ell_n L_{2n}$$



e, se chiamiamo p_n e P_n i perimetri dei poligoni regolari di n lati inscritti e circoscritti, otteniamo

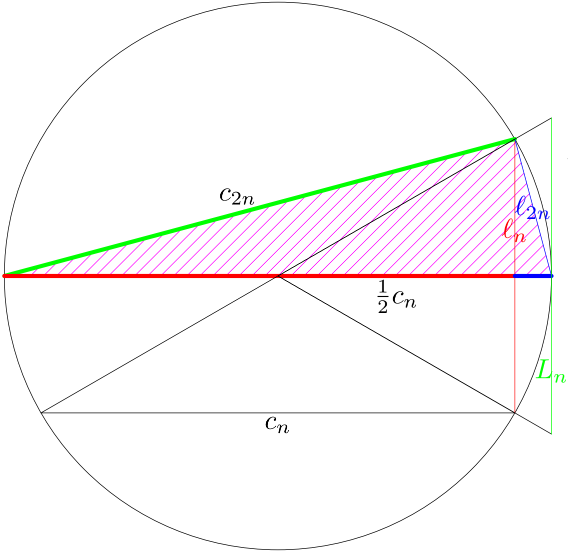
$$4n^2\ell_{2n}^2 = n\ell_n 2nL_{2n}$$

da cui

$$p_{2n}^2 = p_n P_{2n}$$

ed infine

$$p_{2n} = \sqrt{p_n P_{2n}}$$



Per il Teorema di Talete applicato al triangolo tratteggiato in Figura si ha

$$c_{2n}^2 = 2(1 + \frac{1}{2}c_n) = 2 + c_n$$

e, dalle

$$\frac{2\ell_{2n}}{L_{2n}} = c_{2n} = \frac{\ell_n}{\ell_{2n}}$$

$$\frac{2\ell_n}{L_{2n}} = \frac{2\ell_{2n}}{L_{2n}}\frac{\ell_n}{\ell_{2n}} = c_{2n}c_{2n} = c_{2n}^2 = 2 + c_n = 2 + \frac{2\ell_n}{L_n}$$

Dalla precedente uguaglianza si ha:

$$\frac{2\ell_n}{L_{2n}} = 2 + \frac{2\ell_n}{L_n}$$

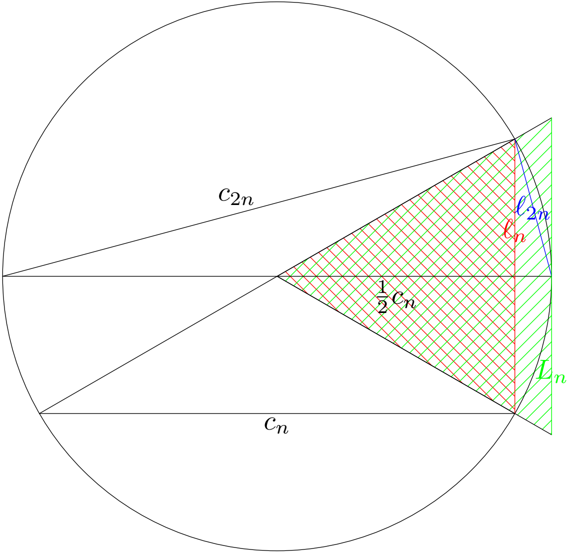
$$\frac{1}{L_{2n}} = \frac{1}{\ell_n} + \frac{1}{L_n} \quad \text{e} \quad L_{2n} = \frac{\ell_n L_n}{L_n + \ell_n}$$

$$2nL_{2n} = \frac{2n\ell_n nL_n}{nL_n + n\ell_n} \quad \text{cioè} \quad P_{2n} = \frac{2p_n P_n}{p_n + P_n}$$

Pertanto avremo che

$$\begin{cases} p_{2n} = \sqrt{p_n P_{2n}} \\ P_{2n} = \frac{2p_n P_n}{p_n + P_n} \end{cases}$$

Le stesse regole di ricorrenza valgono anche per le aree a_n ed A_n dei poligoni regolari inscritti e circoscritti all circonferenza unitaria.



Dalla $\frac{\ell_n}{L_n} = \frac{1}{2}c_n$ si ha

$$\frac{\ell_n}{c_n} = \frac{1}{2}L_n$$

$$A_n = n\frac{L_n}{2} = n\frac{\ell_n}{c_n}$$

$$a_n = n\frac{\ell_n}{2}\frac{c_n}{2}$$

$$a_{2n} = \frac{n\ell_n}{2} = \frac{2a_n}{c_n}$$

Pertanto

$$a_{2n}^2 = \frac{2a_n}{c_n}\frac{n\ell_n}{2} = a_n\frac{n\ell_n}{c_n} = a_n\frac{nL_n}{2} = a_nA_n$$

Inoltre poichè $L_{2n} = \frac{\ell_n L_n}{L_n+\ell_n}$
Otteniamo

$$nL_{2n} = \frac{n\ell_n nL_n}{nL_n + n\ell_n} \qquad \text{da cui} \qquad A_{2n} = \frac{2a_{2n}2A_n}{2A_n + 2a_{2n}} = \frac{2a_{2n}A_n}{A_n + a_{2n}}$$

Pertanto avremo che

$$\begin{cases} a_{2n} = \sqrt{a_nA_n} \\ A_{2n} = \frac{2a_{2n}A_n}{a_{2n}+A_n} \end{cases}$$

La successione (p_n, P_n) e la successione (a_n, A_n) sono definite quindi da legg1 di ricorrenza molto simili che coinvolgono la media geometrica e la media armonica.

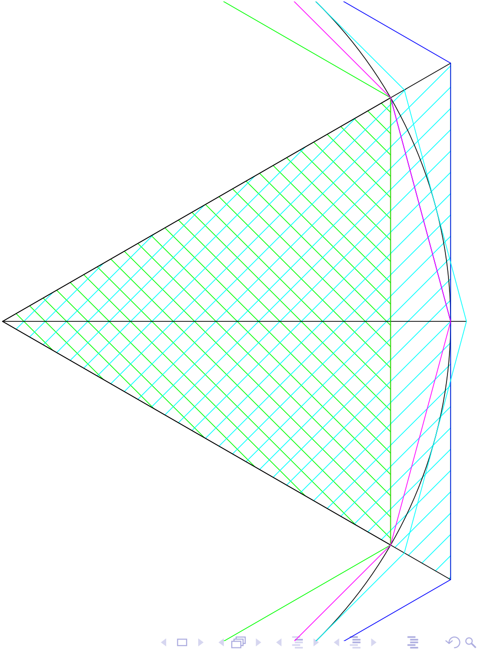
$$\begin{cases} p_{2n} = \sqrt{p_n P_{2n}} \\ P_{2n} = \frac{2p_n P_n}{p_n + P_n} \end{cases} \qquad \begin{cases} a_{2n} = \sqrt{a_n A_n} \\ A_{2n} = \frac{2a_{2n} A_n}{a_{2n} + A_n} \end{cases}$$

con

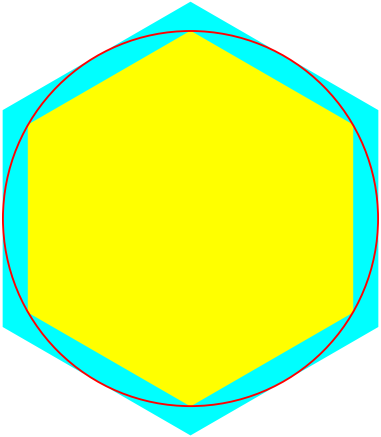
$$(p_0, P_0) = \left(3\sqrt{3}, 6\sqrt{3}\right) \qquad (a_0, A_0) = \left(\frac{3\sqrt{3}}{4}, \frac{3\sqrt{3}}{2}\right)$$

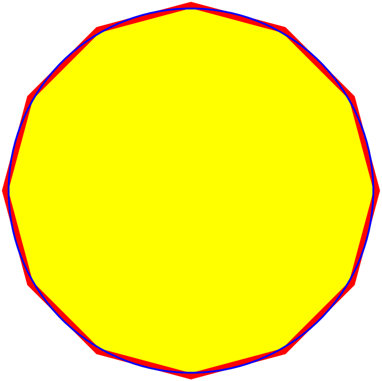
Studiamo ad esempio (p_n, P_n)

È evidente che p_n è crescente e limitata superiormente e che P_n è decrescente e limitata inferiormente;



Inoltre è evidente che $P_n - p_n$ tende a zero.





Ne possiamo quindi dedurre che sia P_n che p_n tendono ad un medesimo valore che definiamo π e che rappresenta l'area del cerchio o la lunghezza della circonferenza unitaria.

Ne deduciamo che $p_n \to \alpha$ e $P_n \to \beta$ con $\alpha, \beta \in \mathbb{R}$.

Passando al limite nelle relazioni di ricorrenza si ottiene

$$\begin{cases} \alpha^2 = \alpha\beta \\ \beta = \frac{2\alpha\beta}{\alpha+\beta} \Longrightarrow \beta^2 + \alpha\beta = 2\alpha\beta \Longrightarrow \beta^2 = \alpha\beta \end{cases}$$

Ne segue subito che $\alpha = \beta$

E possiamo definire il valore comune π

Si calcola che

$$p_{48} = \frac{288}{(6+4\sqrt{3})\sqrt{12\sqrt{2}\sqrt{\frac{\sqrt{3}}{6+4\sqrt{3}}}+\frac{48\sqrt{3}}{6+4\sqrt{3}}}} \approx 6.278700407$$

e

$$P_{48} = \frac{1152\sqrt{2}\sqrt{\frac{\sqrt{3}}{6+4\sqrt{3}}}\sqrt{3}}{(6+4\sqrt{3})(12\sqrt{2}\sqrt{\frac{\sqrt{3}}{6+4\sqrt{3}}}+\frac{48\sqrt{3}}{6+4\sqrt{3}})} \approx 6.292172427$$

La tabella mostra i valori di p_n e di P_n per $n = 3 * 2^k$ con $k = 1 \cdots 10$

k	$3 \cdot 2^k$	p_n	P_n	Errore
0	3	5.196152424	10.39230485	5.196152424
1	6	6	6.928203232	.928203232
2	12	6.211657082	6.430780622	.219123540
3	24	6.265257227	6.319319887	.54062660e-1
4	48	6.278700407	6.292172427	.13472020e-1
5	96	6.282063902	6.285429205	.3365303e-2
6	192	6.282904952	6.283746105	.841153e-3
7	384	6.283115212	6.283325488	.210276e-3
8	768	6.283167780	6.283220353	.52573e-4
9	1536	6.283180936	6.283194070	.13134e-4
10	3072	6.283184205	6.283187491	.3286e-5
11	6144	6.283185052	6.283185858	.806e-6
12	12288	6.283185254	6.283185421	.167e-6

Usualmente si indica con π la lunghezza di una semicirconferenza di raggio 1; si ha con 500 cifre decimali esatte:

$\pi = 3.$
1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679
8214808651328230664709384460955058223172535940812848111745028410270193852110555964462294895493038196
4428810975665933446128475648233786783165271201909145648566923460348610454326648213393607260249141273
7245870066063155881748815209209628292540917153643678925903600113305305488204665213841469519415116094
3305727036575959195309218611738193261179310511854807446237996274956735188575272489122793818301194913